Solids Summer 2002 Final Exam Equations

KML 8/2/02

Axial: $\sigma = \frac{P}{A}$ $\quad \varepsilon = \frac{\delta L}{L}$

Hooke's Law for axial stress $\sigma = E \cdot \varepsilon$ Poisson's Ratio $\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{ax}}$

Average shear stress $\tau = \frac{V}{A}$

Average bearing stress $\sigma_b = \frac{P}{A_b}$ Hooke's Law in shear $\tau = G\,\gamma$

Safety factor $= \frac{\text{failure stress}}{\text{allowable stress}}$ $\quad n_{ult} = \frac{\sigma_{ult}}{\sigma_{all}}$ $\quad n_y = \frac{\sigma_y}{\sigma_{all}}$ $\quad n_{sy} = \frac{\tau_y}{\tau_{all}}$ $\quad$ MS = n -1

Axial deflection $\delta = \frac{P\,L}{A\,E}$ Axial stiffness $k = \frac{A\,E}{L}$

Thermal deflection $\delta L_{th} = L \cdot \alpha \cdot \Delta T$ Thermal stress $\sigma_{th} = E\,\alpha\,\Delta T$

Thermal statically indeterminant $\Sigma \delta L_{th} + \sum \frac{R \cdot L_i}{A_i \cdot E_i} = 0$

Normal stress concentration $\sigma_{max} = K \cdot \sigma_{nom}$

Max torsional stress $\tau_{max} = \frac{T \cdot r}{I_p}$ Shear stress at any radius, *r*,: $\tau = \frac{T}{I_p} \cdot \rho$

Angle of twist $\phi = \frac{T \cdot L}{G \cdot I_p}$ Torsional stiffness $k_T = \frac{G \cdot I_p}{L}$, $\quad T = k_T \cdot \phi$

Torsional shear strain $\gamma_{max} = \frac{r\,\phi}{L}$ Solid shaft $I_p = \frac{\pi \cdot d^4}{32} = \frac{\pi \cdot r^4}{2}$ $\quad \tau_{max} = \frac{16 \cdot T}{\pi \cdot d^3}$

Annulus $I_p = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$

Shear stress concentration $\tau_{max} = K \cdot \tau_{nom}$

Power and torque $P = T\omega$ $\quad \omega = 2\pi f$ $\quad \omega = \frac{\pi\,RPM}{30}$ $\quad P_{hp} = \frac{(T\ \text{ft} \cdot \text{lbs}) \cdot RPM}{5252}$

Bending stress $\sigma_x = -\frac{M_z \cdot y}{I_z}$

Solid rectangular beams $I = \frac{bh^3}{12}$, $\quad \sigma_{max} = \frac{6\,M}{b\,h^2}$ Solid circular beams $I = \frac{\pi d^4}{64}$, $\quad \sigma_{max} = \frac{32\,M}{\pi\,d^3}$

Parallel axis theorem $I_z = \bar{I}_z + A \cdot d_{zz}^2$ Centroid: $\bar{y} = \dfrac{\sum(\bar{y}_i \cdot A_i)}{\sum A_i}$

Widen material2: $\sigma_{x1} = -\dfrac{M\,y}{I_T}$ $\sigma_{x2} = -\dfrac{M\,y}{I_T} \cdot n$ Narrow material1: $\sigma_{x1} = -\dfrac{M\,y}{n \cdot I_T}$ $\sigma_{x2} = -\dfrac{M\,y}{I_T}$ $n = \dfrac{E_2}{E_1}$

Stress from moment and axial force load $\sigma_x = \dfrac{P_x}{A} - \dfrac{M_z \cdot y}{I_z}$ Location of neutral axis $y_0 = \dfrac{P \cdot I}{A \cdot M}$

Transverse shear stress: $\tau = \dfrac{V\,Q}{I\,b}$ $Q = A \cdot \bar{y}_s$

Solid rectangular beam: $\tau_{max} = \dfrac{3\,V}{2\,A}$ $\tau = \dfrac{VQ}{I\,b} = \dfrac{V}{2\,I} \cdot \left(\dfrac{h^2}{4} - y^2\right) = \dfrac{6\,V}{b\,h^3} \cdot \left(\dfrac{h^2}{4} - y^2\right)$

Solid circular beam: $\tau_{max} = \dfrac{4\,V}{3\,A}$

Stress transformations: $\tau_{x1y1} = -\dfrac{\sigma_x - \sigma_y}{2} \cdot \sin(2\theta) + \tau_{xy} \cdot \cos(2\theta)$ $\sigma_{y1} = \sigma_{x1}(\theta + 90°)$

$\sigma_{x1} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2} \cdot \cos(2\theta) + \tau_{xy} \cdot \sin(2\theta)$ $\sigma_{y1} = \dfrac{\sigma_x + \sigma_y}{2} - \dfrac{\sigma_x - \sigma_y}{2} \cdot \cos(2\theta) - \tau_{xy} \cdot \sin(2\theta)$

$\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2}$ $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ $\left|\sin 2\theta_p\right| = \dfrac{\left|\tau_{xy}\right|}{R}$ $\sigma_{1,2} = \sigma_{avg} \pm R$

Plane Stress $\sigma_x = \dfrac{E}{1-\nu^2} \cdot \left(\varepsilon_x + \nu\varepsilon_y\right)$ $\sigma_y = \dfrac{E}{1-\nu^2} \cdot \left(\varepsilon_y + \nu\varepsilon_x\right)$ $\tau_{xy} = G\,\gamma_{xy}$ $G = \dfrac{E}{2 \cdot (1+\nu)}$

$\varepsilon_x = \dfrac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$ $\varepsilon_y = \dfrac{1}{E}\left(\sigma_y - \nu\sigma_x\right)$ $\varepsilon_z = \dfrac{-\nu}{E}\left(\sigma_x + \sigma_y\right)$

Spherical pressure vessels: $\sigma = \dfrac{p\,r}{2\,t}$ $\tau_{max} = \dfrac{p\,r}{4\,t}$

Cylindrical pressure vessels: $\sigma_1 = \dfrac{p\,r}{t}$ $\sigma_2 = \dfrac{p\,r}{2\,t}$ $\tau_{max} = \dfrac{p\,r}{2\,t}$

Deflection charts provided on additional pages.

Buckling: $P_{cr} = \dfrac{\pi^2\,E\,I}{L_e^2}$

Pinned-pinned: $L_e = L$ Fixed-free: $L_e = 2L$ Fixed-fixed: $L_e = 0.5\,L$ Fixed-pinned: $L_e = 0.699\,L$