$$q_{sph} = \frac{4\pi k (T_{s1} - T_{s2})}{1/r_1 + 1/r_2}$$

Conduction

$q = -kA\frac{dT}{dx}$

✓assume
- 1-D steady heat transfer
- no generation
- negligible radiation
- uniform properties

$R_{eq\,lin} = \frac{L}{kA}$ $\quad dx = L \quad dT = T_1 - T_2$

$q'' = -k\frac{dT}{dx}$

$R'_{eq\,rad} = \frac{\ln \frac{r_2}{r_1}}{2\pi k}$

Convection

$q = hA_{\parallel}(T_s - T_\infty)$

$R_{eq\,lin} = \frac{1}{hA}$

$q' = hw(T_s - T_\infty)$; $q'' = h(T_s - T_\infty)$

$R'_{eq\,rad} = \frac{1}{2\pi r h}$

Diffusion equation:

$$\frac{\delta}{\delta x}\left(k\frac{\delta T}{\delta x}\right) + \frac{\delta}{\delta y}\left(k\frac{\delta T}{\delta y}\right) + \frac{\delta}{\delta z}\left(k\frac{\delta T}{\delta z}\right) + \underbrace{\dot{q}}_{\text{generation}} = \rho C_p \frac{\delta T}{\delta t}$$

$\alpha = \frac{k}{\rho C_p}$ $\quad \frac{dT}{dx} = \phi$ if linear $\quad h \doteq \frac{W}{m^2 K}$

$\therefore \frac{d^2T}{dx^2} = 0$ & $T(x) = C_1 x + C_2$ $\quad k \doteq \frac{W}{mK}$

$$\frac{\delta^2 T}{\delta x^2} + \frac{\delta^2 T}{\delta y^2} + \frac{\delta^2 T}{\delta z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\delta T}{\delta t}$$

$r \quad \phi \quad z$

$q \doteq W$

$q' \doteq \frac{W}{m}$

Ohm's Law

$q'' \doteq \frac{W}{m^2}$

$q = \frac{T_1 - T_2}{R_1} = \frac{kA(T_1 - T_2)}{L}$ $\quad r_{cr} = \frac{k_{ins.}}{h}$ $\quad \dot{q} \doteq W/m^3$

Fins

$\theta = T - T_\infty$ $\quad$ infinite fin $\quad mL > 2.65$

$\theta_A = T_A - T_\infty$

$m = \sqrt{\frac{hP}{kA_c}}$ $\quad$ effectiveness $\quad \varepsilon = \frac{q_f}{q_{w/o\,f}}$

surface area of fin

$q_F = \left[\sqrt{hPkA_c}\cdot\theta_b\right] = \sqrt{hPkA_c}(T_b - T_\infty)$ $\quad A_f = 2 \cdot L_c \cdot w$

$\eta_f = \frac{q_F}{hA_F\theta_b}$ $\quad$ * $A_f = 2\cdot\pi(r_{2c}^2 - r_1^2)$

$A_f = L_c \cdot w \cdot 2$ $\quad P = 2\cdot w$ (t negligible)

$\eta_f = mL_c$ $\quad P = 2\pi r$

Radiation

$q' = \frac{q}{w} = \eta h A_f \theta_b$ $\quad A_c = w \cdot t$

$A_c = \pi r^2$

$q_{rad} \propto AT^4$ $\quad$ generation

$q_{rad} = \sigma A T^4$

$q_{rad} = \epsilon F_{12} \sigma A (T_1^4 - T_2^4)$ $\quad T_0 = \frac{\dot{q}L^2}{2k} + T_s$ $\quad \frac{dT}{dx} = -\frac{\dot{q}L}{k}$

emissivity shape factor

$q' = \dot{q}A$

$Bi = \frac{hL_c}{k}$ if $Bi << 0.1$ then 1-D (Lumped capacitance)

$L_c \equiv \frac{V}{A_s}$ rect. plate : $\frac{A(2L)}{2A} = L$ cylinder: $L_c = \frac{\pi r^2 L}{2\pi r L} = \frac{r}{2}$

$Q_{max} = \rho C_p V (T_i - T_\infty)$

$Q = \rho C_p V (T_i - T_\infty)\left[1 - e^{-(t/\tau)}\right]$ & $\tau = \frac{\rho C_p V}{hA}$

$\frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\tau}$

Conduction

$q \propto \frac{dT}{dx}$

$q = -kA_{\perp}\frac{dT}{dx}$ } 1-D, steady heat transfer
no generation, negligible radiation
uniform properties

$q' = -k\,w\,\frac{dT}{dx}$

$q'' = -k\frac{dT}{dx}$

Convection

$q = hA_{\parallel}(T_s - T_{\infty})$

$q' = hw(T_s - T_{\infty})$

$q'' = h(T_s - T_{\infty})$

$q = \frac{(T_s - T_{\infty})}{R_{conv.}} = \frac{(T_s - T_{\infty})}{\frac{1}{hA_{\parallel}}}$

$A_{lin} = L \cdot w \qquad A_{rad} = \pi D \cdot L = 2\pi r L$

$g_c = 32.1740 \frac{lbm \cdot ft}{lbf \cdot s^2}$

thermal diffusivity $\alpha = \frac{k}{\rho c_p}$

$Bi = \frac{hL_c}{k}$ if $Bi << 0.1 \Rightarrow$ 1-D (L.C.) T doesn't vary w/position

$L_c = \frac{V}{A}$; cyl: $L_c = \frac{r}{2}$ plate: $L_c = L$ sphere: $L_c = r/3$

$V_{sphere} = 4/3 \pi r^3$
$A_{sphere} = 4 \pi r^2$

Lumped capacitance

$$Q = \underbrace{\rho C_p V (T_i - T_\infty)}_{Q_{max}} \left[1 - e^{(-t/\tau)}\right]$$

also $Q = \rho C_p V (T_i - T)$

Don't use if Not L.C.

T @ time

$$\tau = \frac{\rho c_p V}{hA}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{(-t/\tau)}$$

$$\frac{Q}{Q_{max}} = 1 - e^{(-t/\tau)}$$

$$Q_{max} = Q_o = \rho V C_p \Delta T$$

$$Q'_{max} = Q'_o = \rho A C_p \Delta T$$

If not Lumped Capacitance use heisler charts

Semi-infinite solids

Case 1: Imposed T

$$\frac{T(x,t) - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$q''_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

case 3: convection

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - e^{\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)} \cdot erfc\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

or chart on pg. 271

$$* \; erfc(w) = 1 - erf(w)$$

Semi-infinite solids Numerical method

Forward Difference (explicit) : $T_m^{p+1} = \left[1 - \frac{2\alpha\Delta t}{(\Delta x)^2}\right] T_m^{\,p} + \frac{\alpha \Delta t}{\Delta x^2}\left(T_{m+1}^{\,p} + T_{m-1}^{\,p}\right)$ (F_o written above $\frac{2\alpha\Delta t}{(\Delta x)^2}$)

if $F_o \geqslant \frac{1}{2}$ then stable

$$F_o = \frac{\alpha \Delta t}{(\Delta x)^2}$$

Interior nodes : $T_m^{p+1} = \left[1 - 2F_o\right] T_m^{\,p} + F_o\left(T_{m+1}^{\,p} + T_{m-1}^{\,p}\right)$

if possible choose $F_o = \frac{1}{2}$

- solve for Δt or Δx

Δt	T_1	T_2	etc.

node exposed to convection : $T_o^{p+1} = (1 - 2F_o - 2F_o B_i) T_o^{\,p} + 2F_o T_1^{\,p} + 2F_o B_i T_\infty$: to be stable $F_o(1 + B_i) \leq \frac{1}{2}$

$\theta^*_{AVG}(x, r, t) = \theta^*(r, t) \cdot \theta^*(x, t)$; $\frac{x}{L} \Rightarrow \frac{\theta}{\theta_i}$ & $\left(\frac{\theta}{\theta_i}\right)_{x_1} \cdot \left(\frac{\theta}{\theta_i}\right)_{x_2} = \left(\frac{\theta}{\theta_i}\right)_{AVG}$

__Heat Exchangers__

$q = UA\Delta T_{Lm}$ multipass (shell & tube) : $q = UAF\Delta T_{Lm}$

$$\Delta T_{Lm} = \left[\frac{(T_{Ho} - T_{Co}) - (T_{Hi} - T_{Ci})}{\ln\left(\frac{T_{Ho} - T_{Co}}{T_{Hi} - T_{Ci}}\right)}\right] : \text{parallel flow}$$

$$\Delta T_{Lm} = \left[\frac{(T_{Ho} - T_{Ci}) - (T_{Hi} - T_{Co})}{\ln\left(\frac{T_{Ho} - T_{Ci}}{T_{Hi} - T_{Co}}\right)}\right] : \text{cross flow}$$

$$Q_{max} = \underbrace{(\dot{m} C_{p_{min}})}_{C} (\Delta T)_{max}$$

$$\mathcal{E} = \frac{Q}{Q_{max}}$$

$$NTU = \frac{AU}{C_{min}}$$

$$C_R = \frac{C_{min}}{C_{max}}$$