$v = \frac{1}{\rho}$

specific weight $\gamma = \rho g$

$SG = \frac{\rho}{\rho_{H_2O @ 4°C}}$

$Re = \frac{\rho V D}{\mu}$

$\mu = \frac{C T^{3/2}}{T + S}$ (gases)

$\mu = D e^{B/T}$ (liquids)

$\tau = \mu \frac{dU}{dy}$

$h = \frac{P_1 - P_2}{\gamma}$

$P = \gamma h + P_o$

$Q = V \cdot A$

$F = P \cdot A$

$\dot{m} = \rho w A = \rho Q$

$w = \frac{Q}{A}$

B (Boyancy) = (sp. wt.)(Vol.$_{H_2O}$ Displaced)

S.T. (surface tension) = σ(boundary length)

ρ_{H_2O} @ 4°C = 1000 kg/m³

$\frac{\rho}{\mu} = \frac{1}{\nu}$

Resultant Forces

$F_R = \gamma h_c A$

$y_R = \frac{I_{xc}}{y_c A} + y_c$

$x_R = \frac{I_{xyc}}{y_c A} + x_c$

$y_T A_T = y_1 A_1 + y_2 A_2 \ldots$

P_{Atm} @ S.L. = 14.696 lb/in² abs.

$\rho_{H_2O} = \begin{cases} 1.94 \ \text{slug}/\text{ft}^3 \\ 999 \ \text{kg}/\text{m}^3 \end{cases}$ $\quad \gamma_{H_2O} = \begin{cases} 62.4 \ \text{lb}/\text{ft}^3 \\ 9.80 \ \text{kN}/\text{m}^3 \end{cases}$

$\rho_{Air} = \begin{cases} 2.58 \times 10^{-3} \ \text{slug}/\text{ft}^3 \\ 1.23 \times 10^{0} \ \text{kg}/\text{m}^3 \end{cases}$ $\quad \gamma_{AIR} = \begin{cases} 7.65 \times 10^{-2} \ \text{lb}/\text{ft}^3 \\ 1.2 \times 10^{1} \ \text{N}/\text{m}^3 \end{cases}$

B.E.; $P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$

$P_{atm} = 14.696 \frac{lb}{in^2}$ abs.

$P_{atm} = 1.013 \times 10^{-5}\ N/m^2$ abs.

relative roughness = ϵ

Flow in round pipe

$V = \frac{Q}{A}$

$Re \leq 2100$ laminar

$Re \geq 4000$ turbulent

$Re = \frac{\rho V D}{\mu}$

$2100 < Re < 4000$ transitional (treat as laminar)

$\frac{l_e}{D} = 0.06\, Re$ laminar

$f = \frac{64}{Re}$ (laminar)

$\frac{l_e}{D} = 4.4\, (Re)^{1/6}$ turbulent

$\frac{\Delta P}{l} = \frac{2\tau}{r}$

$$\tau = \frac{2\,\tau_w\, r}{D}$$

τ_w: shear @ wall; r: instantaneous radius; D: diameter of pipe

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_L$$

$$\Delta P = \frac{4 l\, \tau_w}{D}$$

$$P_1 - P_2 = \gamma(z_2 - z_1) + \gamma h_L = \gamma(z_2 - z_1) + f\frac{l}{D}\frac{\gamma V^2}{2g}$$

vel. $\rightarrow u = -\left(\frac{\Delta P}{4\mu l}\right) r^2 + C_1$ & $C_1 = \frac{\Delta P}{16 \mu l} D^2$

$$u(r) = \left(\frac{\Delta P D^2}{16 \mu l}\right)\left(1 - \left(\frac{2r}{D}\right)^2\right) = V_c\left(1 - \left(\frac{2r}{D}\right)^2\right)$$

V_c: vel. @ centerline

$$u(r) = \frac{\tau_w D}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right] \quad \text{where } R = \frac{D}{2}$$

$$Q = \frac{\pi R^2 V_c}{2} = \frac{\pi D^4 \Delta P}{128 \mu l}$$

$V_c = 2V$

$$h_L = f\frac{l}{D}\frac{V^2}{2g} \quad \begin{pmatrix} D_1 = D_2 \\ V_1 = V_2 \\ z_1 = z_2 \\ \alpha_1 = \alpha_2 \end{pmatrix}$$

$Re = \frac{VD}{\nu}$

$h_L = k_L \frac{V^2}{2g}$, $\Delta P = k_L \cdot \frac{1}{2} \cdot \rho V^2$

$$\boxed{\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \text{ eqn for Moody Chart (Turbulent Only)}}$$

$$D = \int p \cos\theta \, dA + \int \tau_w \sin\theta \, dA$$
$$L = -\int p \sin\theta \, dA + \int \tau_w \cos\theta \, dA$$

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$$

$$Re = \frac{\rho U l}{\mu} = \frac{U l}{\nu}$$

$Re > 100$: neglect viscosity (outside B.L.)

$Re < 1$: neglect inertia

$$M_a = \frac{U}{c}$$

$$F_r = \frac{U}{\sqrt{g l}}$$

flat plate

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

$$D_f = \tfrac{1}{2}\rho U^2 b l \, C_{Df}$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$D = \rho b U^2 \theta$ valid for laminar or turbulent flow

$\tau_w = \rho U^2 \frac{d\theta}{dx}$ (Lam or Turb)

$D = \rho b U^2 \theta$ (Lam. or Turb)

$$C_{Df} = \frac{1.328}{\sqrt{Re_l}} \qquad Re_l = \frac{U l}{\nu} \qquad Re_x = \frac{U x}{\nu}$$

$\parallel$ to mot. $\Leftarrow D = C_D \tfrac{1}{2}\rho U^2 A$

$\perp$ to mot. $\Leftarrow L = C_L \tfrac{1}{2}\rho U^2 A$

English	ρ (slugs/ft³)	γ (lb/ft³)	μ (lb·s/ft²)	ν (ft²/s)
Water	1.94	62.4	2.34 E-5	1.21 E-5
Air	2.38 E-3	7.65 E-2	3.74 E-7	1.57 E-4

Metric	ρ (kg/m³)	γ (kN/m³)	μ (N·s/m²)	ν (m²/s)
water	999	9.8	1.12 E-3	1.12 E-6
air	1.23 E+0	1.20 E+1	1.79 E-5	1.46 E-5

cons of mass $$\frac{\delta}{\delta t}\overbrace{\int_{CV} \rho\, d\forall}^{m} + \int_{CS} \rho(\overset{\text{velocity}}{V}\cdot\hat{n})\,dA = 0$$

$Q = \overset{\text{vel.}}{V}\cdot A$

$\dot{m} = \rho Q = \rho A V$ (V: $\perp$ Velocity)

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$\rho = \frac{P}{RT}$

avg. vel. $\bar{V} = \frac{\int_A \rho(V\cdot\hat{n})\,dA}{\rho A}$

$+$ = in
$-$ = out

$$\text{Force}\quad \frac{\delta}{\delta t}\overbrace{\int_{CV} V\rho\, d\forall}^{\text{momentum } mV} + \int_{CS} V\rho(V\cdot\hat{n})\,dA = \Sigma F$$

$\rho V^2 A$

$-$ = in
$+$ = out

$V = w + u$ (V: abs. vel. of H_2O; w: rel. vel. of H_2O; u: vel. nozzle)

$$\text{moment}\quad \frac{\delta}{\delta t}\int_{CV}(r\times V)\rho\, d\forall + \int_{CS} V\rho\,(r\times V)\cdot\hat{n}\,dA = \Sigma(r\times F)$$

$$\text{Energy}\quad \frac{\delta}{\delta t}\underbrace{\int_{CV} e\rho\,\forall}_{\text{time rate of change of total stored energy}} + \int_{CS} e\rho V\cdot\hat{n}\,dA = (\dot{Q}_{\underset{in}{net}} + \dot{W}_{\underset{in}{net}})_{CV}$$

$Q_{in} \equiv$ pos.
$W_{in} \equiv$ pos.

energy per unit mass $e = u + \frac{V^2}{2} + gz$

$\dot{W}_{\text{normal stress}} =$

$\dot{W}_{shaft} = \omega\cdot T_{shaft}$ (ω: avg vel.; T: torque)

$$\frac{\delta}{\delta t}\int_{CV} e\rho\, d\forall + \int_{CS}\left(u + \frac{P}{\rho} + \frac{V^2}{2} + gz\right)\underbrace{\rho V\cdot\hat{n}\,dA}_{\text{becomes } \dot{m} \text{ when uniform flow}} = \dot{Q}_{\underset{in}{net}} + \dot{W}_{\underset{in}{net}}$$

$$\Sigma\dot{m}\left[u + \frac{P}{\rho} + \frac{V^2}{2} + gz\right] = \dot{Q}_{net} + \dot{W}_{net}$$

$$\Sigma\left(\dot{m}\left[h + \frac{V^2}{2} + gz\right]\right) = \dot{Q}_{net} + \dot{W}_{net}$$

1D steady flow

if u, P/ρ, $\frac{V^2}{2}$, gz are all uniformly distributed over the flow cross sectional area.

$$\dot{m}\left[u_o - u_{in} + \left(\frac{P}{\rho}\right)_o - \left(\frac{P}{\rho}\right)_{in} + \frac{V_o^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{\underset{in}{net}} + \dot{W}_{\underset{net\,in}{shaft}}$$

$h = u + \frac{P}{\rho}$; $\frac{1}{\rho} = v$

(1D steadly in mean flow)

$$\dot{m}\left[h_{out} - h_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{\underset{in}{net}} + \dot{W}_{\underset{net\,in}{shaft}}$$

$u_{out} - u_{in} - q_{net}$ = loss of energy

$$\therefore \quad \frac{P_{out}}{\rho} + \frac{V_{out}^2}{2} + g z_{out} = \frac{P_{in}}{\rho} + \frac{V_{in}^2}{2} + g z - \text{loss} + w_{\substack{shaft \\ net\,in}}$$

$\text{loss} \doteq \frac{ft \cdot lb}{slug} \doteq \frac{ft^2}{s^2}$ $\qquad \text{loss} = k_L \frac{V_2^2}{2}$? where did this equation come from?

head $\equiv$ energy per unit weight

$$\frac{P_{out}}{\gamma} + \frac{V_{out}^2}{2g} + z_{out} = \frac{P_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} + h_s - h_L$$

head loss

$$h_s \equiv \frac{\dot{W}_{shaft}}{\dot{m} g} = \frac{\dot{W}_{shaft}}{\gamma Q} = \frac{w_{shaft}}{g}$$

not enthalpy

$$h_{Turbine} = -(h_s + h_L)_{Turb.}$$
$$h_{pump} = (h_s - h_L)_{pump}$$

Units of loss

$\frac{ft \cdot lb}{slug}$,

$1\ hp = 2545\ \frac{BTU}{hr}$ $\qquad 1\ BTU = 778.17\ ft \cdot lbf$

$1\ hp = 550\ \frac{ft \cdot lbf}{s}$

$1\ hp = 0.7457\ kW$

$1\ atm \begin{cases} 1.01325\ bar \\ 14.696\ psi \end{cases}$