$x_H = Ae^{pt}$

$\tau = \frac{1}{a} \quad (\dot{x} + ax = u)$

$D_c = 2\sqrt{km}$

$D_c = 2M\omega_n \quad \omega_n \sim \text{rad/s}$

$\frac{D}{2m} = \xi\omega_n \quad D_c \sim \frac{\text{lb-sec}}{\text{ft}}$

$m\ddot{x} + k\dot{x} + Dx = 0 \quad \omega_d \sim \text{rad/s}$

$\omega_n^2 = \frac{k}{m}$

$\frac{D}{m} = 2\xi\omega_n$

$\omega_d = \sqrt{1-\xi^2} \cdot \omega_n$

$\omega_n \tau = 2\pi$

$m_{spring} = 1/3 m$

$\frac{dx}{dt} + ax = u$

$x_H = Ae^{-at}$

$x_p = \cancel{c}$

$\dot{x}_p = 0$

$0 + a\cancel{c} = u \Rightarrow \cancel{c} = \frac{u}{a}$

$x = Ae^{-at} + \frac{u}{a}$

apply I.C. $x(0) = 0$

$0 = Ae^{-a(0)} + \frac{u}{a}$

$A = -\frac{u}{a}$

$x = \frac{u}{a}(1 - e^{-at})$

sky diver $+\downarrow \sum F_y = m\dot{v}$

mg

m

DV^2

$mg - DV^2 = m\dot{v}$

$m\dot{v} + DV^2 = mg$

$m\dot{v} = 0$ @ terminal vel.

$V = \sqrt{\frac{mg}{D}}$

Eulers Identities

$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots$

$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!}$

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{-j\theta} = \cos\theta - j\sin\theta$

$\mathcal{L} f(t) = F(s) = \int_0^\infty e^{-st} \cdot f(t) \cdot dt$

$\mathcal{L}\, x = x(s)$

$\mathcal{L}\, \cancel{c} = \cancel{c}/s$

$\frac{d}{dx}[\sin u] = (\cos u)u'$

$\frac{d}{dx}[\cos u] = -(\sin u)u'$

$\frac{DW}{k} = 2\left(\overset{\xi}{\frac{D}{2\sqrt{km}}}\right)\left(\overset{1/\omega_n}{\frac{\sqrt{km}}{k}}\right) \cdot \omega \quad ?$

$\frac{Dj\omega}{k} \cdot \frac{2\sqrt{km}}{2\sqrt{km}} = \overbrace{\frac{D}{2\sqrt{km}}}^{\xi} \cdot \overbrace{\frac{2\sqrt{km}}{k}}^{2\,1/\omega_n} \cdot \omega j$

$\frac{dh}{dt} + .001h = .01$

$h_H = Ae^{-.001t}$

$h_p = \cancel{c}$

$\dot{h}_p = 0 \qquad 0 + .001c = .01$

$c = 10$

$h = Ae^{-.001t} + 10$

$h(0) = 0$

$0 = Ae^{-.001(0)} + 10$

$A = -10$

$h = -10e^{-.001t} + 10$

Partial fraction Expansion

short cut method

$$\frac{1}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$$ mult. by $(s+2)$ let $s \to -2$

$$\frac{1}{s+4} = A + \frac{B(s+2)}{s+4} \Rightarrow A = 1/2$$

real roots

$$X(s) = \frac{2}{s^2(s+2)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2}$$

mult. by $s^2(s+2)$

add like s terms

(j) has complex roots

$$X(s) = \frac{10}{s(s^2+4s+13)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+13}$$

method to avoid complex roots — mult. by $s(s^2+4s+13)$

add like terms

(j) complex roots

$$\frac{10}{s(s^2+4s+13)} = \frac{A_1}{s} + \frac{A_2}{s+2-3j} + \frac{A_3}{s+2+3j}$$

repeated real roots

$$\frac{1}{s(s^2+2s+1)} = \frac{1}{s(s+1)(s+1)} = \frac{1}{s(s+1)^2}$$

$$\frac{1}{s(s^2+2s+1)} = \frac{A_1}{s} + \frac{A_2}{(s+1)^2} + \frac{A_3}{(s+1)}$$

complex roots short cut

$$\frac{10}{s(s^2+4s+13)} = \frac{A_1}{s} + \frac{A_2}{s+2-3j} + \frac{A_3}{s+2+3j}$$

1. mult. by each den.
2. let that den. $\to 0$

$$A = \frac{1+3j}{2-4j} = a + bj$$

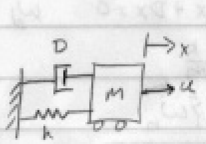
$$A = \frac{(1+3j)(2+4j)}{(2-4j)(2+4j)} = \frac{2+10j-12}{2^2+4^2}$$

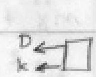
$$\frac{dx}{dt} + ax = u \qquad x(0) = 0$$

$$s \cdot X(s) - x(0) + a\,X(s) = \frac{u}{s}$$

$$X(s)(s+a) = \frac{u}{s}$$

$$X(s) = \frac{u}{s(s+a)}$$





$$\Sigma F = m\ddot{x}$$

$$-D\dot{x} - kx + u = m\ddot{x}$$

$$m\ddot{x} + D\dot{x} + kx = u$$

$$\ddot{x} + \frac{D}{m}\dot{x} + \frac{k}{m}x = \frac{u}{m}$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{u}{m}$$

$$\frac{j}{j} = -1$$

$$\frac{1}{j} = -j$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$s = j\omega$ method

$m\ddot{x} + D\dot{x} + kx = F_0 \cos(\omega t)$

$X(s)[ms^2 + Ds + k] = F_0(s)$ ← may be mult by $\cancel{\ }$

$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + Ds + k}$

let $s = j\omega$

$\frac{X(j\omega)}{F(j\omega)} = \frac{X}{F} e^{j\phi} = \frac{1}{-m\omega^2 + Dj\omega + k}$

$\Sigma M_o = J\ddot{\theta}$

$T = J\ddot{\theta}$

$T = -mgl\theta$ ← small angle $\sin\theta \approx \theta$

$J_o\ddot{\theta} + mgl\theta = 0$

$\ddot{\theta} + \frac{mgl}{J_o}\theta = 0$

$\omega_n = \frac{2\pi}{\tau} \Rightarrow$ where $\tau = \frac{30\text{ sec}}{24\text{ cycles}} = 1.25\ \text{s/cyc.}$

$\omega_n = 5.0265\ \text{rad/s}$

$\omega_n^2 = \frac{mgl}{J_o} = \frac{wl}{J_o}$

$J_o = 16.62\ \text{lb in s}^2$

$J_{cg} = J_o - ml^2$

$= 16.62 - \frac{70}{32.2 \cdot 12} \cdot 6^2$

$J_{cg} = 10.1\ \text{lb in s}^2$

$D_c = 2\sqrt{km} = 2m\omega_n$ (rad/s)

$\frac{D_c}{2m} = \xi\omega_n$

$\omega_n^2 = \frac{k}{m}$

$\frac{D}{m} = 2\xi\omega_n$

$\omega_d = \sqrt{1-\xi^2} \cdot \omega_n$

$\omega_n \tau = 2\pi$

$\xi = \frac{D}{D_c}$

Logarithmic Decrement

$\delta = \ln\left(\frac{X_o}{X_i}\right) = \xi\omega_n\tau_d$

$= \xi\omega_n\left(\frac{2\pi}{\omega_d}\right) = \frac{2\pi\xi\omega_n}{\sqrt{1-\xi^2} \cdot \omega_n}$

$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$ *

$\omega_d^* \tau_d = 2\pi$

$\omega_d = \sqrt{1-\xi^2} \cdot \omega_n^*$

$\omega_n^2 = \frac{k}{m}$ *

$D_c = 2\sqrt{km}$

$\frac{D}{D_c} = \xi$

$x_{ss}(t) = X\cos(\omega t - \phi)$

$X = \frac{\frac{F_o}{k}\cos(\omega t - \phi)}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$ $\qquad \phi = \tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$

Method from D.E.: $\ddot{x} + \frac{D}{m}\dot{x} + \frac{k}{m}x = F_o \cos\omega t$

e.g. $1\ddot{x} + 10\dot{x} + 400x = 100\cos 4t$

$\ddot{x} + 10/1\,\dot{x} + 400/1\,x = (100\cos 4t)/1$

$\omega_n^2 \Rightarrow \omega_n = 20 \qquad \omega$

$D_c = 2\sqrt{km} = 40$

$\xi = \frac{D}{D_c} = 0.25 \qquad \frac{\omega}{\omega_n} = 0.2$

$\phi = \frac{2(.25)(.2)}{1-(.2)^2} = 5.947°$

$\phi = 0.1038$ rad

$X = \frac{100/400}{\sqrt{[1-0.2^2]^2 + [2(.25)(.2)]^2}} = 0.2590$

Electrical Systems

$V = IR$

$P = I^2R = \frac{V^2}{R}$

Inductor $V = L\frac{dI}{dt} = Ls$

cap: $V = \frac{1}{C}\int I\,dt = \frac{1}{Cs}$

parraled

$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$

KVL L1 ⇒ V_1

KVL L2 ⇒ $I(s)$

assumes output leads input

$x_{ss}(t) = X\cos\omega t + \phi = Xe^{j\omega t} \cdot e^{j\phi}$

$\dot{x}_{ss}(t) = j\omega Xe^{j\omega t} \cdot e^{j\phi}$

e.g. $\dot{x} + ax = b\sin(\omega t)$

after above derivation substitute

$j\omega X\cancel{e^{j\omega t}} \cdot e^{j\phi} + aX\cancel{e^{j\omega t}} \cdot e^{j\phi} = b\cancel{e^{j\omega t}}$

$Xe^{j\phi}[j\omega + a] = b$

$Xe^{j\phi} = \frac{b}{j\omega + a} \quad \therefore \quad X = \frac{\sqrt{0^2 + b^2}}{\sqrt{\omega^2 + a^2}} = \frac{b}{\sqrt{\omega^2 + a^2}}$

$\phi = \tan^{-1}\left(\frac{0}{b}\right) - \tan^{-1}\frac{\omega}{a} = -\tan^{-1}\frac{\omega}{a}$

Classical Method

$x_p(\sin bx \text{ or } \cos bx) = A\sin bx + B\cos bx$

$x_p(e^{ax}) = Ae^{ax}$

$A\cos\omega t + B\sin\omega t = \sqrt{A^2 + B^2}\cos(\omega t - \phi)$ ← or sin if $f(x) = b\sin(\omega t)$

$\phi = \tan^{-1}\left(\frac{B}{A}\right)$

$x_{ss} = x_p \qquad x_H$ = transient response (dies out over time)

$\dot{x} + ax = b\cos\omega t \qquad x(0) = 0$

$x(t) = Ae^{-at} + B\cos(\omega t) + A\sin(\omega t)$

$\dot{x}_p = -B\omega\sin(\omega t) + C\omega\cos(\omega t)$

$-B\omega\sin(\omega t) + C\omega\cos\omega t + aB\cos(\omega t) + aC\sin(\omega t) = b\cos(\omega t)$

$\Rightarrow C = \frac{b}{\omega + a^2/\omega} = \frac{b\omega}{\omega^2 + a^2} \Rightarrow B = \frac{ab}{\omega^2 + a^2}$

$x(t) = Ae^{-at} + \frac{ab}{a^2 + \omega^2}\cos(\omega t) + \frac{b\omega}{a^2 + \omega^2}\sin(\omega t)$

apply $x(0) = 0 \Rightarrow A = -\frac{ab}{\omega^2 + a^2}$

$x(t) = \frac{ab}{a^2 + \omega^2}\cos(\omega t) + \frac{b\omega}{a^2 + \omega^2}\sin(\omega t) - \overbrace{\frac{ab}{\omega^2 + a^2}e^{-at}}^{\text{transient as } t \to \infty,\ e^{-at} \to 0}$

$x_{ss}(t) = \frac{ab}{a^2 + \omega^2}\cos(\omega t) + \frac{b\omega}{a^2 + \omega^2}\sin(\omega t)$

apply

$x_{ss}(t) = \sqrt{\left(\frac{ab}{a^2 + \omega^2}\right)^2 + \left(\frac{b\omega}{a^2 + \omega^2}\right)^2}\cos\left(\omega t - \frac{\omega}{a}\right)$

$x_{ss}(t) = X\cos(\omega t + \phi) \qquad y = Ye^{j\omega t} \qquad x = Xe^{j\omega t} \cdot e^{j\phi}$

$y(t) = Y\cos(\omega t) \qquad \dot{y} = j\omega Ye^{j\omega t} \qquad \dot{x} = j\omega Xe^{j\omega t} \cdot e^{j\phi}$

$\ddot{x} = -\omega^2 Xe^{j\omega t} \cdot e^{j\phi}$

obtained D.E.:

$m\ddot{x} + D\dot{x} + kx = D\dot{y} + ky$

$-m\omega^2 X\cancel{e^{j\omega t}}e^{j\phi} + j\omega DX\cancel{e^{j\omega t}}e^{j\phi} + kX\cancel{e^{j\omega t}}e^{j\phi} = Dj\omega Y\cancel{e^{j\omega t}} + kY\cancel{e^{j\omega t}}$

$Xe^{j\phi}[-m\omega^2 + j\omega D + k] = Y[Dj\omega + k]$

$\frac{X}{Y}e^{j\phi} = \frac{Dj\omega + k}{-m\omega^2 + j\omega D + k} = \frac{Dj\omega/k + 1}{-\frac{m}{k}\omega^2 + \frac{j\omega D}{k} + 1}$

because $\frac{\sqrt{km}}{k} = \sqrt{\frac{km}{k^2}} = \frac{1}{\omega_n}$

$\left\{\frac{Dj\omega}{k} \cdot \frac{2\sqrt{km}}{2\sqrt{km}} = \frac{D}{2\sqrt{km}} \cdot \frac{2\sqrt{km}}{k} \cdot j\omega = \xi \cdot 2 \cdot \frac{1}{\omega_n} \cdot j\omega\right\}$

$\frac{X}{Y}e^{j\phi} = \frac{2\xi\frac{\omega}{\omega_n}j + 1}{2\xi\frac{\omega}{\omega_n}j + 1 - \left(\frac{\omega}{\omega_n}\right)^2} \quad \Big| \quad \frac{X}{Y} = \frac{\sqrt{j^2 + R^2}}{\sqrt{j^2 + R^2}}$

$90° = \pi/2$

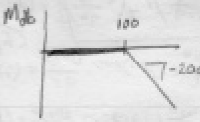
$\phi = \tan^{-1}\left(\frac{j}{R}\right)_n - \tan^{-1}\left(\frac{j}{R}\right)_D$

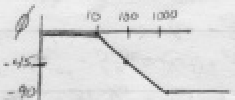
$G(s) = \frac{1}{.01s+1}$

$G(j\omega) = \frac{1}{.01j\omega+1}$

$|G(j\omega)| = \frac{1}{\sqrt{(.01\omega)^2+1}}$

$\phi = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{.01\omega}{1}\right)$





$\omega_c = 100$

$\omega \ll 100$ $|G(j\omega)| = 1$

$M_{db} = 20\log(1) = 0$

$\omega >> 100$ $|G(j\omega)| = \frac{1}{.01\omega}$

$M_{db} = 20\log(1) - 20\log(.01\omega)$

for $\omega = 100$ $M_{db} = 0$

$\omega = 1000$ $M_{db} = -20db$

$\omega = 10{,}000$ $M_{db} = -40db$

$\phi = -\tan^{-1}(.01\omega)$

$\omega \ll 100$; $\phi = 0$

$\omega = 100$; $\phi = -45°$

$\omega >> 100$; $\phi = -90°$

$G(s) = 0.1s+1$

$G(j\omega) = 0.1j\omega+1$

$|G(j\omega)| = \sqrt{(0.1\omega)^2+1}$

$\phi = \tan^{-1}\left(\frac{0.1\omega}{1}\right)$

$\omega_c = 10$

$\omega \ll 10$ $|G(j\omega)| = 1$

$M_{db} = 0$

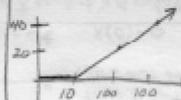
$\omega >> 10$ $|G(j\omega)| = 0.1\omega$

$M_{db} = 20\log(0.1\omega)$

$\omega = 10$: $M_{db} = 0$

$\omega = 100$: $M_{db} = 20$

$\omega = 1000$: $M_{db} = 40$

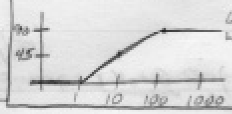


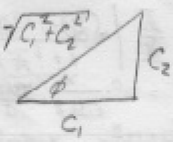
$\phi = \tan^{-1}(0.1\omega)$

$\omega \ll 10$: $\phi = 0$

$\omega = 10$: $\phi = 45$

$\omega >> 10$: $\phi = 90$





$\cos\phi = \frac{C_1}{\sqrt{C_1^2+C_2^2}}$

$\sin\phi = \frac{C_2}{\sqrt{C_1^2+C_2^2}}$

$G(s) = \frac{10(s+100)}{s(s+10)} \cdot \frac{1/10}{1/10}$

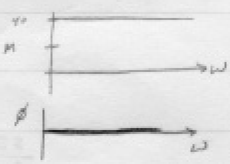
$G(s) = \frac{s+100}{s(T_1s+1)}$ $T = 0.1$

$G(s) = \frac{100(T_2s+1)}{s(T_1s+1)}$ $T_2 = 0.01$

$G(s) = \frac{100(0.01s+1)}{s(0.1s+1)}$

Factor 100 ; $M_{db} = 20\log 100 = 40dB$

$\phi = \tan^{-1}\left(\frac{0}{100}\right) = 0$



Factor $\frac{1}{s}$: $G(s) = \frac{1}{s}$

$G(j\omega) = 1/j\omega$

$|G(j\omega)| = \frac{1}{\omega}$

$M_{db} = 20\log\frac{1}{\omega}$

$= 20\log 1 - 20\log\omega$

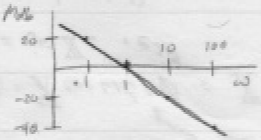
$\omega = 1$ $M_{db} = 0$

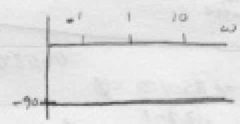
$\omega = 10$ $M_{db} = -20$

$\omega = 100$ $M_{db} = -40$

$\phi = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega}{0}\right)$

$\phi = -90°$





Factor $(.01s+1)$: $G(s) = \frac{j\omega}{100} + 1$

$G(j\omega) = \sqrt{\left(\frac{\omega}{100}\right)^2+1}$

$\omega \ll 100$ $|G(j\omega)| = 1 = 0dB$

$\omega >> 100$ $|G(j\omega)| = \frac{\omega}{100}$

$M_{db} = 20\log 100 - 20\log 100 = 0$

$= 20\log 1000 - 20\log 100 = 20$

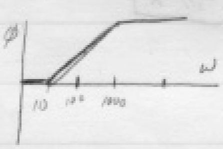
$= 20\log 10{,}000 - 20\log 100 = 40$

$\phi = \tan^{-1}\left(\frac{.01\omega}{1}\right) - \tan^{-1}\left(\frac{0}{1}\right)$

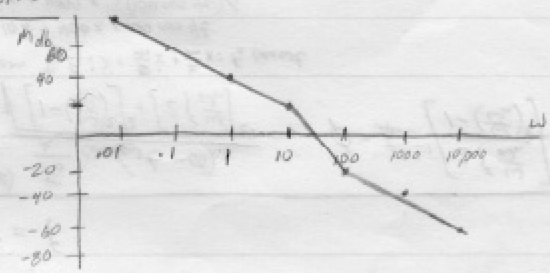
$\phi(100) = 45°$

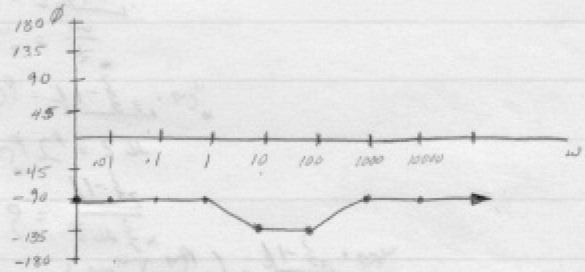
$\phi(\omega >> 100) = 90°$

$\phi(\omega \ll 100) = 0$



Composite





Factor $\left(\frac{1}{.1s+1}\right)$: $G(j\omega) = \frac{1}{0.1j\omega+1}$

$|G(j\omega)| = \frac{1}{\sqrt{(0.1\omega)^2+1}}$

$\omega_c = 10$

$\omega \ll 10$ $G(j\omega) = 1 \Rightarrow M_{dB} = 0$

$\omega >> 10$ $G(j\omega) = 1/0.1\omega$

$\omega = 10$ $M_{dB} = 20\log 1 - 20\log 0.1 - 20\log\omega = 0$

$\omega = 100$ $M_{dB} = -20$

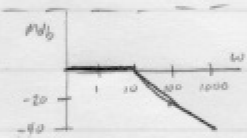
$\omega = 1000$ $M_{dB} = -40$

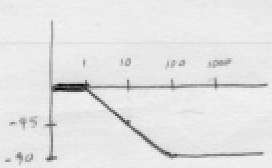
$\phi = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{0.1\omega}{1}\right)$

$\omega = 1$ $\phi = 0$

$\omega = 10$ $\phi = -45$

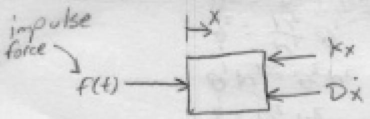
$\omega = 100$ $\phi = -90$





IMPULSE

$f(t)=1$
$F(s)=1$



$I = F\cdot\Delta t$ (force; time force applied)

$$-kx - D\dot{x} + f(t) = m\ddot{x}$$

$$m\ddot{x} + D\dot{x} + kx = f(t)$$

L.T.:

$$s^2 m X(s) - sm\cancel{x(0)}^{0} - m\cancel{\frac{dx}{dt}\Big|_{t=0}}^{0} + sDX(s) - D\cancel{x(0)}^{0} + kx(s) = F\cdot\Delta t$$

$$X(s)(s^2 m + sD + k) = F\cdot\Delta t$$

$$X(s) = \frac{F\cdot\Delta t}{s^2 m + sD + k}$$

same prob.

$$\int_0^t f(t) = mV_0$$

$$F\cdot t = mV_0$$

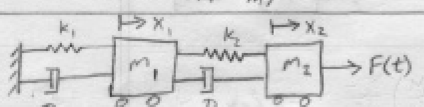
$$V_0 = \frac{m}{f(t)} = \dot{x}_0$$

L.T.:

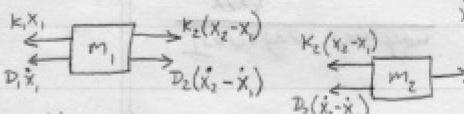
$$s^2 m X(s) - sm\cancel{x(0)}^{0} - m\frac{dx_0}{dt}\Big|_{t=0} + sDX(s) - D\cancel{x(0)}^{0} + kx(s) = 0$$

$$X(s)\left[s^2 m + sD + k\right] = m\dot{x}_0$$

$$X(s) = \frac{\cancel{m}\dot{x}_0}{\cancel{m}(s^2 + \frac{D}{m}s + \frac{k}{m})}$$

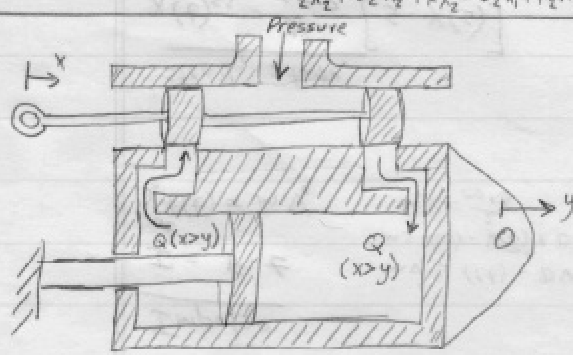


assume
$x_1 > 0$
$\dot{x}_1 > 0$
$x_2 > x_1$
$\dot{x}_2 > \dot{x}_1$



$$m_1\ddot{x}_1 + (D_1+D_2)\dot{x}_1 + (k_1+k_2)x_1 = D\dot{x}_2 + k_2 x_2$$

$$m_2\ddot{x}_2 + D_2\dot{x}_2 + kx_2 = D_2\dot{x}_1 + k_2 x_1 + f(t)$$



$$Q = k_v(x-y)$$

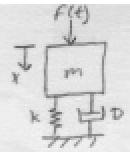
$$\dot{y}A = Q$$

$$\dot{y}A = k_v(x-y)$$

$$\dot{y}A + k_v y = k_v x$$

$$sy(s)A + k_v y(s) = k_v X(s)$$

$$\text{T.F.} \rightarrow \frac{y(s)}{X(s)} = \frac{k_v}{sA + k_v} = \frac{k_v/A}{s + k_v/A}$$



$f(t) = u$ (unit step)

$$F(s) = \frac{u}{s}$$

Transfer Function / steady state

$$m\ddot{x} + D\dot{x} + kx = f(t)$$

$$x(0)=0,\ \dot{x}(0)=0$$

L.T.

$$ms^2 x(s) - m\cancel{sx(0)}^{0} - m\cancel{\dot{x}(0)}^{0}\big|_{t=0} + DsX(s) - D\cancel{x(0)}^{0} + kx(s) = \frac{u}{s}$$

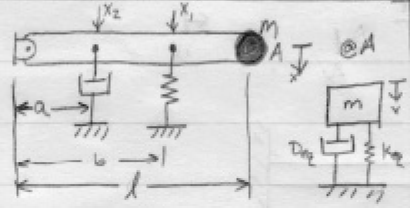
$$X(s)\left[ms^2 + Ds + k\right] = \frac{u}{s}$$

$$X(s) = \frac{u}{s(ms^2+Ds+k)}$$

$$x(t)_{ss} = \lim_{s\to 0}\left[s\cdot X(s)\right]$$

$$x(t)_{ss} = \frac{\cancel{s}\cdot u}{\cancel{s}(ms^2+Ds+k)}\Big|_{s=0}$$

$$x(t)_{ss} = \frac{u}{k}$$

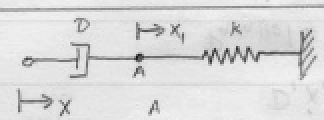


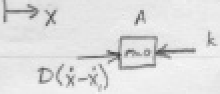
$$k_{eq@A} = \left(\frac{b}{l}\right)^2 k$$

$$D_{eq@A} = \left(\frac{a}{l}\right)^2 D$$

$$D_c@A = 2m_{eq}\sqrt{\frac{k_{eq}}{m_{eq}}} = 2\left(\frac{b}{l}\right)\sqrt{km}$$

$$D_c@B = \left(\frac{l}{a}\right)^2\cdot D_c@A = \frac{2bl}{a^2}\sqrt{km}$$





$$D\dot{x} - D\dot{x}_1 - kx_1 = 0$$

$$D\dot{x}_1 + kx_1 = D\dot{x}$$

$$\dot{x}_1 + \frac{k}{D}x_1 = \dot{x}$$

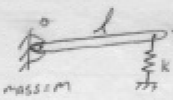
Grocery scale — x @ platform facing Down

$$\frac{1}{2}\left[m_p + \frac{m_k}{3} + m_d\right]\dot{x}^2 + \frac{1}{2}J_{G1}\dot{\theta}_1^2 + \frac{1}{2}J_{G2}\dot{\theta}_2^2$$

$$\theta_1 R = x \qquad \dot{\theta}_1 = \frac{\dot{x}}{R} \qquad \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} \Rightarrow \dot{\theta}_2 = \dot{\theta}_1\frac{N_1}{N_2} = \frac{\dot{x}}{R}\frac{N_1}{N_2}$$

$$\cancel{\tfrac{1}{2}}\left[m_p + \frac{m_k}{3} + m_d\right]\dot{x}^2 + \cancel{\tfrac{1}{2}}J_{G1}\frac{\dot{x}^2}{R^2} + \cancel{\tfrac{1}{2}}J_{G2}\frac{\dot{x}^2}{R^2}\cdot\frac{N_1^2}{N_2^2} = \cancel{\tfrac{1}{2}}m_{eq}\dot{x}^2$$

$$m_{eq} = m_p + \frac{m_k}{3} + m_d + \frac{J_{G1}}{R^2} + \frac{J_{G2}}{R^2}\left(\frac{N_1}{N_2}\right)^2$$



equiv. sys @ O

$$\frac{1}{2}J_0\dot{\theta}^2 = \frac{1}{2}J_{eq}\dot{\theta}_0^2$$

$$\cancel{\tfrac{1}{2}}\left(\frac{ml^3}{3}\right)\dot{\theta}_0^2 = \cancel{\tfrac{1}{2}}J_{eq}\dot{\theta}_0^2$$

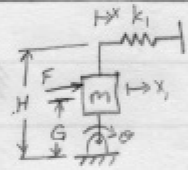
$$J_{eq} = \frac{ml^3}{3}$$

$$\frac{1}{2}kx^2 = \frac{1}{2}k_{Teq}\theta_0^2$$

$$\theta_0\cdot l = x$$

$$\cancel{\tfrac{1}{2}}kl^2\theta_0^2 = \cancel{\tfrac{1}{2}}k_{Teq}\theta_0^2$$

$$k_{Teq} = kl^2$$



F=?

$$\frac{1}{2}k_1 x^2 = \frac{1}{2}k_{eq}x_1^2$$

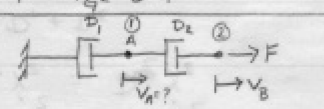
$$x = \theta H \qquad x_1 = \theta G$$

$$x_1 = x\frac{H}{G}$$

$$k_{eq} = k_1\frac{H^2}{G^2}$$

$$F = k_{eq}\cdot x_1$$

$$F = k_1\frac{H^2}{G^2}\cdot\theta G$$



$$D_{eq}@B: \quad \frac{D_1\cdot D_2}{D_1+D_2}$$

$$F = D_{eq}\cdot V_B = \frac{D_1\cdot D_2}{D_1+D_2}\cdot V_B$$

$$F = D_1\cdot V_A$$

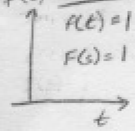
$$\therefore \frac{D_1\cdot D_2}{D_1+D_2}\cdot V_B = D_1\cdot V_A$$

$$V_A = \frac{D_2}{D_1+D_2}\cdot V_B$$

Impulse

$f(t)=1$

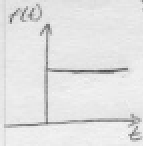
$F(s)=1$



Unit step

$f(t)=1$

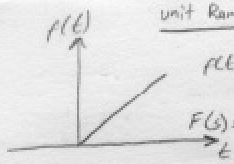
$F(s)=\frac{1}{s}$



unit Ramp

$f(t)=t$

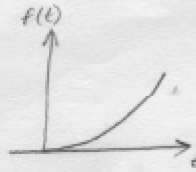
$F(s)=\frac{1}{s^2}$



unit parabolic

$f(t)=\frac{t^2}{2}$

$F(s)=\frac{1}{s^3}$



$$\omega_n^2 = \frac{k}{m}$$

$$2\xi\omega_n = \frac{D}{m}$$

$$D_c = 2m\omega_n$$

$$\omega_d = \sqrt{1-\xi^2}\,\omega_n$$

Impulse

$$I = F\cdot t$$

$$\int_0^{\Delta t} f(t) = mV_0$$

$$m\dot{v} = f(t) - DV$$

$$msV(s) - V(0)^0 + DV(s) = 2$$

$$V(s) = \frac{2}{ms+D}$$

transform laplace of input

steady state

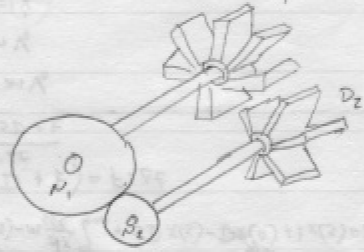
$$x(t)_{ss} = \lim_{s\to 0}\left[s\cdot X(s)\right]$$

Transfer Function

$$\frac{X(s)}{F(s)}$$

Force Systems

1. Find Equivalent system
2. Sum Forces



$$D_1\dot{\theta}_1^2 + D_2\dot{\theta}_2^2 = D_{eq}\dot{\theta}_1^2$$

$$x = \theta_1 N_1$$

$$x = \theta_2 N_2$$

$$\dot{\theta}_1 N_1 = \dot{\theta}_2 N_2$$

$$\dot{\theta}_2 = \frac{N_1}{N_2}\dot{\theta}_1$$

$$D_{eq} = D_1 + D_2\left(\frac{N_1}{N_2}\right)^2$$

Mass Equiv. systems

$$\tfrac{1}{2}m_1V_1^2 + \tfrac{1}{2}m_2V_2^2 = \tfrac{1}{2}m_{eq}V_2^2$$

$$m_{spring} = \tfrac{1}{3}\cdot m$$

$$\tfrac{1}{2}J_1\dot{\theta}_1^2 + \tfrac{1}{2}J_2\dot{\theta}_2^2 = \tfrac{1}{2}J_{eq}\dot{\theta}_2^2$$

Spring Equiv. systems

$$\tfrac{1}{2}k_1X_1^2 + \tfrac{1}{2}k_2X_2^2 = \tfrac{1}{2}k_{eq}X_2^2$$

$$\tfrac{1}{2}k_{T_1}\theta_1^2 + \tfrac{1}{2}k_T\theta_2^2 = \tfrac{1}{2}k_{Teq}\theta_2^2$$

Parallel: $k_{eq} = k_1 + k_2$ (have same motion)

series: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$ (have same force)

$$\therefore k_{eq} = \frac{k_1k_2}{k_1+k_2}$$

Damper Equiv. Systems

$$D_1\dot{X}_1^2 + D_2\dot{X}_2^2 = D_{eq}\dot{X}_2^2$$

Parallel: $D_{eq} = D_1 + D_2$

series: $\frac{1}{D_{eq}} = \frac{1}{D_1} + \frac{1}{D_2}$

$$D_{eq} = \frac{D_1D_2}{D_1+D_2}$$

$$D\dot{x}_2 + kx_2 = kx_1$$