Properties of Equality

$a = a$, Reflexive

If $a = b$, then $b = a$; Symmetric

If $a = b$ and $b = c$, then $a = c$; Transitive

If $a = b$, then a may be replaced by b in any expression that involves a. Substitution

Properties of Inequalities

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ac < bc$.

If $a < b$ and $c < 0$, then $ac > bc$.

Properties of Equations

If $a = b$, then $a + c = b + c$.

If $a = b$, then $ac = bc$.

Properties of Absolute Value Inequalities

$|x| < c$ $(c \geq 0)$ if and only if $-c < x < c$.

$|x| > c$ $(c \geq 0)$ if and only if either $x > c$ or $x < -c$.

Properties of Exponents

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(a^m b^n)^p = a^{mp} b^{np}$$

$$\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$$

$$b^{-p} = \frac{1}{b^p}$$

Properties of Radicals

$$(\sqrt[n]{b})^m = \sqrt[n]{b^m} = b^{m/n}$$

$$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}, \qquad b \neq 0$$

$$\sqrt[m]{\sqrt[n]{b}} = \sqrt[mn]{b}$$

Properties of Logarithms

$y = \log_b x$ if and only if $b^y = x$

$$\log_b b = 1 \qquad \log_b 1 = 0$$

$$\log_b (b)^p = p \qquad b^{\log_b p} = p$$

$$\log_b (MN) = \log_b M + \log_b N$$

$$\log_b (M/N) = \log_b M - \log_b N$$

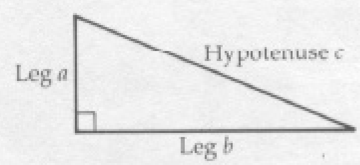
$$\log_b M^p = p \log_b M$$

$$\log x = \log_{10} x \qquad \ln x = \log_e x$$

Important Theorems

Pythagorean Theorem

$c^2 = a^2 + b^2$



Remainder Theorem

If a polynomial $P(x)$ is divided by $x - c$, then the remainder is $P(c)$.

Factor Theorem

A polynomial $P(x)$ has a factor $(x - c)$ if and only if $P(c) = 0$.

Fundamental Theorem of Algebra

If P is a polynomial of degree $n \geq 1$ with complex coefficients, then P has at least one complex zero.

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{k} a^{n-k} y^k + \cdots + b^n$$

Important Formulas

Slope m of a line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Slope-intercept form of a line with slope m and y-intercept b

$$y = mx + b$$

Point-slope formula for a line with slope m passing through $P_1(x_1, y_1)$

$$y - y_1 = m(x - x_1)$$

Quadratic Formula

If $a \neq 0$, the solutions of $ax^2 + bx + c = 0$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

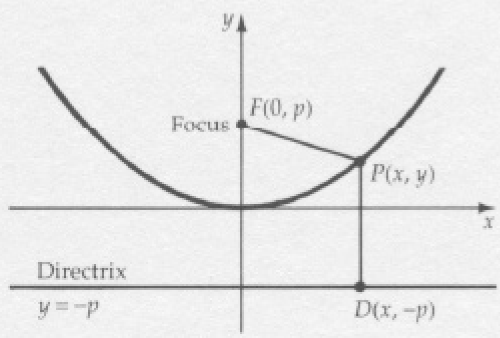
Distance Formula

The distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

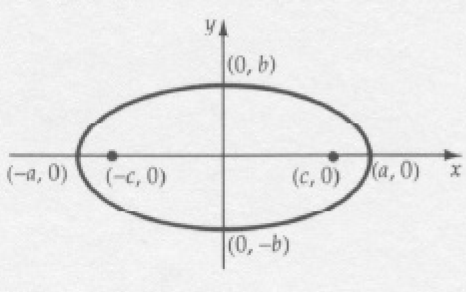
$$d(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Formulas for the conic sections

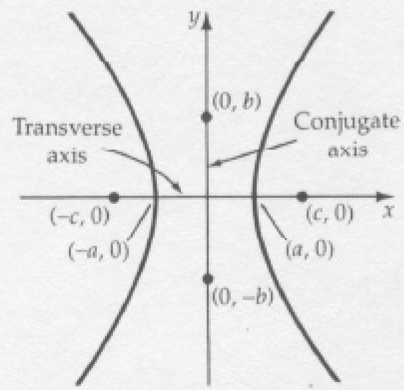
Parabola: $x^2 = 4py$



Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



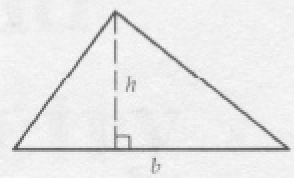
Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Formulas for area *A*, circumference *C*, and volume *V*.

Triangle

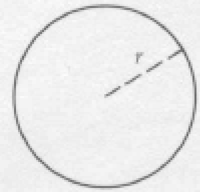
$$A = \frac{1}{2}bh$$



Circle

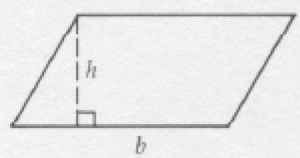
$$A = \pi r^2$$

$$C = 2\pi r$$



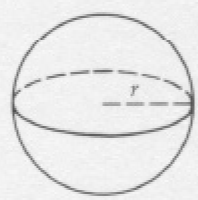
Parallelogram

$$A = bh$$



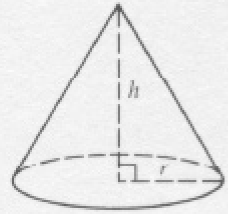
Sphere

$$V = \frac{4}{3}\pi r^3$$



Right circular cone

$$V = \frac{1}{3}\pi r^2 h$$



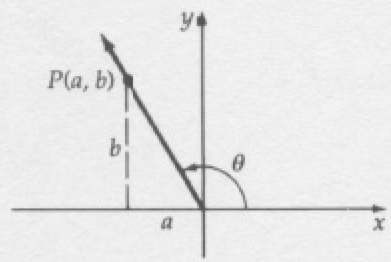
Definitions of Trigonometric Functions

Angle θ

$\sin\theta = \frac{b}{r} \qquad \csc\theta = \frac{r}{b}$

$\cos\theta = \frac{a}{r} \qquad \sec\theta = \frac{r}{a}$

$\tan\theta = \frac{b}{a} \qquad \cot\theta = \frac{a}{b}$

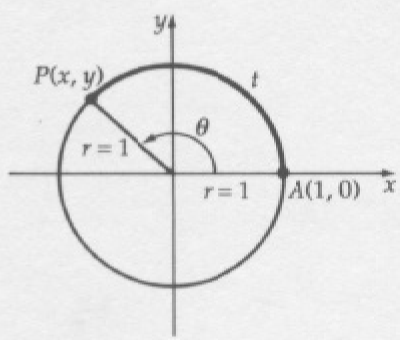


Real Number t (Circular Functions)

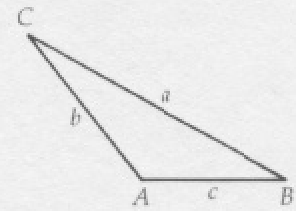
$\sin t = y \qquad \csc t = \frac{1}{y}$

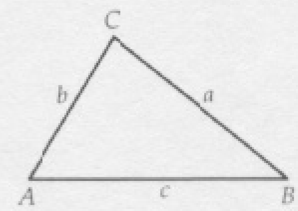
$\cos t = x \qquad \sec t = \frac{1}{x}$

$\tan t = \frac{y}{x} \qquad \cot t = \frac{x}{y}$



Oblique Triangles





Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Area K of the Triangle

$$K = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A$$

Heron's Formula

$$K = s\sqrt{(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{a+b+c}{2}$$

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Fundamental Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Formulas for Negatives

$\sin(-t) = -\sin t$ $\quad\cos(-t) = \cos t$

$\tan(-t) = -\tan t$

Sum of Two Angle Identities

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

Difference of Two Angle Identities

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double-angle Identities

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Half-angle Identities

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2} \qquad \cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\tan^2\alpha = \frac{1-\cos 2\alpha}{1+\cos 2\alpha}$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

Product-to-Sum Identities

$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$

$$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$

Sum-to-Product Identities

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$